HYPER VERSION

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Locally localized gravity

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Abstract: We study the fluctuation spectrum of linearized gravity around non-fine-tuned branes. We focus on the case of an AdS_4 brane in AdS_5. In this case, for small cosmological constant, the warp factor near the brane is essentially that of a Minkowski brane. However, far from the brane, the metric differs substantially. The space includes the AdS_5 boundary, so it has infinite volume. Nonetheless, for sufficiently small AdS_4 cosmological constant, there is a bound state graviton in the theory, and four-dimensional gravity is reproduced. However, it is a massive bound state that plays the role of the four-dimensional graviton.

Keywords: Field Theories in Higher Dimensions, AdS/CFT Correspondance.

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Contents

1. Introduction

The realization that even spin 2 excitations can be localized by a gravitating brane [1] suggests an alternative to compactification, since one has an infinite extra dimension but nonetheless sees four-dimensional gravity at low scales. However, it seemed a critical aspect of this proposal was the finite "volume" of the extra-dimensional space since that is what permitted a normalizable zero-mode and a finite value of the four-dimensional Planck scale. However, it would be perplexing if this finite volume is truly essential, since we should expect physics to be local, so it would be strange if there were strong dependence on the warp factor far from the brane on which gravity potentially localizes.

For this reason, it is very interesting to study an AdS_4 brane in AdS_5, in which the tuning of cosmological terms required for a flat brane is modified. The metric is known in this case, and it is a warped geometry in which the warp factor blows up at large distance from the brane, rather than asymptoting to zero as it does in the flat brane case. Therefore, the geometry includes the boundary of AdS_5 and the volume of the extra-dimensional space is infinite. Clearly, the zero mode, corresponding to a perturbation whose wavefunction in the 5th dimension is proportional to the warp factor, is non normalizable.

Nonetheless, since such a theory can be obtained from the flat brane scenario through an arbitrarily small reduction of the cosmological term on the brane, it is difficult to believe that there is no bound state graviton to generate four-dimensional physics. Indeed, we find that four-dimensional gravity is reproduced, due to a very light, but massive bound state Kaluza-Klein mode of the graviton. Of course the gravity we reproduce is AdS gravity, so the cosmological constant would have to be sufficiently small so that physics looks minkowskian within the horizon size.

This is an important result, as it demonstrates that localization is a local phenomenon, as indeed it must be. A warped geometry in a local region can be sufficient to generate four-dimensional gravity independently of the behavior of the higher-dimensional geometry far away. Although we explicitly analyze the AdS_4 brane in AdS_5, we expect this result is far more general.

For those familiar with the c-theorem in five dimensions, it might be surprising that we can have a warp factor that localizes gravity near the brane, but then rises again. However, we will show this is consistent in AdS space.

This theory probably has interesting implications from the AdS/CFT vantage point, since it includes both a Planck brane and the AdS_5 boundary, which we briefly comment on here.

Furthermore, this theory gives rise to four-dimensional gravitational physics on a brane embedded into an AdS_5 bulk that asymptotes to the boundary of AdS and not the interior as its Minkowski or dS cousin, thereby avoiding the no-go theorems about smooth RS branes and supergravity and possibly faciliating a realization of localized gravity in that context.

In section 2, we review the background geometry for a single brane with a non-finetuned tension, that is with an effective cosmological constant. In section 3 we will discuss the fluctuation spectrum around such a background. While in the dS case one still has a massless bound graviton, we will show that in the AdS case, the massless bound state is lost. However one does find an almost massless graviton bound on the brane, which we refer to as the almost zero mode. In section 4, we argue that four-dimensional gravity is correctly reproduced. We conclude in the final section.

2. The background

2.1 Action and background solution

as in the original setup [1], the action we consider is just that of 5d gravity with a negative cosmological constant $\Lambda^{5d} = -3/L^2$ coupled to a brane of tension λ:

$$S = \int d^5x \sqrt{g}\left[-\frac{1}{4}R - \Lambda^{5d}\right] - \lambda \int d^4x\, dr \sqrt{|\det g_{ij}|}\,\delta(r)\,, \tag{2.1}$$

where g_{ij} is the metric induced on the brane by the ambient metric $g_{\mu\nu}$.

We use the ansatz for the solution to be a warped product with warp factor $A(r)$,

$$ds^2 = e^{2A(r)}\bar{g}_{ij}dx^i dx^j - dr^2\,, \tag{2.2}$$

allowing for the 4d metric to be Minkowski, de Sitter or anti-de Sitter with the 4d cosmological constant Λ being zero, positive or negative, respectively following the conventions of [2].

The solutions to Einstein's equations[1] are [2, 5, 6, 7]:

$$\begin{aligned}
dS_4 : A &= \log\left(\sqrt{\Lambda}L\sinh\frac{c-|r|}{L}\right), \qquad \lambda = \frac{3}{L}\coth\frac{c}{L}\,,\\
M_4 : A &= \frac{c-|r|}{L}\,, \qquad \lambda = \frac{3}{L}\,,\\
AdS_4 : A &= \log\left(\sqrt{-\Lambda}L\cosh\frac{c-|r|}{L}\right), \qquad \lambda = \frac{3}{L}\tanh\frac{c}{L}\,.
\end{aligned} \tag{2.3}$$

The geometry of the curved solutions is characterized by a parameter c, which is determined in terms of the brane tension. In the dS case c is the distance between the brane and the horizon, whereas in the AdS case c is the distance to the turn around point in the warp factor. As is well known, in order to have a Minkowski solution, one has to fine-tune λ relative to L; in our conventions $\lambda = 3/L$. Since $\coth(x) > 1$ and $\tanh(x) < 1$ for any x, we see that, $\lambda > 3/L$ implies that the effective 4d cosmological constant is positive and $\lambda < 3/L$ implies a negative 4d cosmological constant. The 4d cosmological constant in 4d Planck units is given by

$$4\pi \int dr\, e^{2\left(A(r)-\frac{1}{2}\ln|\Lambda|\right)} \tag{2.4}$$

and is hence determined by c alone, that is by the mismatch of brane tension and bulk cosmological constant. Since only the combination Λe^{2A} appears in the equations of motion, not both, Λ and A, will be determined independently. One can use this freedom to set $A(0) = 0$, as in [1]. This way the cosmological constant becomes

$$\Lambda_{\mathrm{dS}} = \frac{1}{L^2\sinh^2\frac{c}{L}}\,, \qquad \Lambda_{\mathrm{AdS}} = \frac{-1}{L^2\cosh^2\frac{c}{L}}\,. \tag{2.5}$$

[1] Actually these solutions have been known and studied before, [3, 4]. It is in the references we give that they have been reconsidered in the context of localized gravity.

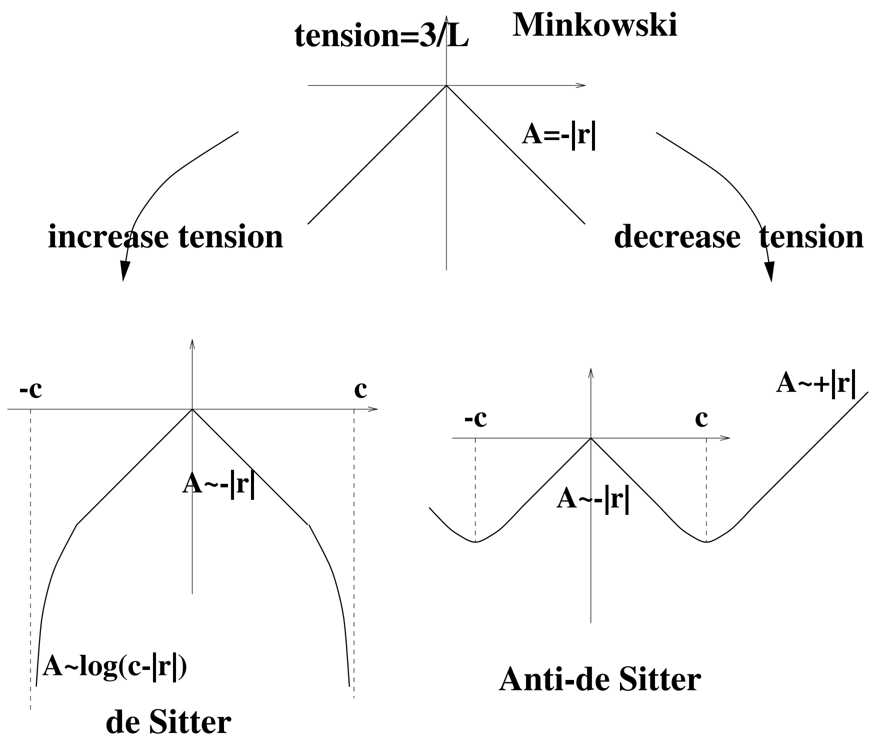

Figure 1: The behavior of the warp-factor for $\Lambda = -1$, 0 and 1.

Using the value of c determined by the jump equations one finds that the 4d cosmological constant is indeed only given by the detuning

$$M = \frac{\lambda L}{3} \tag{2.6}$$

of the brane tension without any exponential surpression:

$$\Lambda_{\mathrm{dS}} = \frac{1}{L^2}(M^2 - 1)\,, \qquad \Lambda_{\mathrm{AdS}} = \frac{1}{L^2}(1 - M^2)\,. \tag{2.7}$$

The general behavior of the three solutions is sketched in figure 1. In the fine-tuned Minkowski solution, the warp factor is just $A = -|r|$ and the graviton is marginally bound on the brane; that is, it is a bound state at threshold. The onset of the continuum is not separated by a mass gap. Increasing the tension the backreaction on the warp factor is even stronger and even though $A(r)$ starts out linear in r, as in the Minkowski case, it goes to $-\infty$ at a finite value of r, at $r = c$. $A(r)$ diverges as $\log(c - |r|)$, but all components of the curvature tensor stay finite. Since the bulk solution is nothing but AdS_5, the coordinate singularity we see in this particular coordinate system is just a horizon. Gravity is still trapped in the usual way. Since we increased the tension of the brane, one should expect that the marginal bound state becomes a true bound state. Formally one can even take the limit where the bulk cosmological constant vanishes altogether, $L \to \infty$, in which case one still traps gravity. The curvature vanishes identically in the bulk. The coordinate singularity reduces to the Rindler horizon of an accelerated observer in flat Minkowski space; see [5].

If we however decrease the tension of the brane as compared to the fine-tuned value, the backreaction on the warp factor becomes "weaker". Even though it still starts out as $A \sim -|r|$ close to the brane it reaches a minimum at a $r = c$ and then turns around and runs toward $+\infty$ as $A \sim +|r|$, approaching the boundary of AdS_5 and not the interior. So even though close to the brane it still looks like we trap gravity in the standard way, much of the amplitude for the zero mode state is concentrated near the boundary of the bulk space. However, since the cause of losing the bound graviton arises only far from the brane, at what should correspond to low energies, one should expect localization. In the next section, we will show that four-dimensional gravity is reproduced, first by a propagator analysis, and subsequently, by showing the interesting mode analysis that gives the same result. The authors of [2] gave the warp factor but did not study this physics, presumably due to the absence of phenomenologists.

2.2 The modified C-theorem

In the language of AdS flows the positive energy condition has been rephrased as the so called "c-theorem" [8, 9] which asserts that $A'' \leq 0$. This is one of the reasons that it was believed that localization of gravity on a positive tension brane goes together with a warpfactor that tends asymptotically to the AdS horizon, since it can not turn around. In the solution we are considering A'' is obviously positive even though our brane has positive tension, so it does not violate positivity of stress energy. Let us see how this is consistent with the c-theorem.

The derivation of the c-theorem required Lorentz invariance and hence is only valid for Minkowski solutions. Even the AdS_4 brane violates $A'' \leq 0$, since

$$A''(r) = \frac{1}{L^2} \frac{1}{\cosh^2 \frac{c \pm |r|}{L}} \tag{2.8}$$

is positive everywhere in the bulk. Using the fact that the energy momentum tensor supporting our geometry has to be of the form

$$T^\alpha_\beta = \mathrm{diag}\{\rho, -p_1, -p_2, -p_3, -p_4\} \tag{2.9}$$

what is called the "weaker energy condition" in [8]

$$T_{\alpha\beta} \xi^\alpha \xi^\beta \geq 0 \,, \tag{2.10}$$

where ξ^α is an arbitrary null vector translates into the condition

$$\rho + p_i \geq 0 \,. \tag{2.11}$$

It is easy to see that energy momentum tensors supporting solutions of the general form of a warped product based on 4d Minkowski, de Sitter or anti-de Sitter space will automatically satisfy the conditions for $i = 1, 2, 3$. Since

$$-3A'' - 3\Lambda e^{-2A} = R^0_0 - R^5_5 = G^0_0 - G^5_5 = \left(T^0_0 - T^5_5\right) \geq 0 \tag{2.12}$$

the constraint on A'' from the weaker energy condition reads

$$A'' \leq -\Lambda e^{-2A} \,. \tag{2.13}$$

For positive Λ this implies an even stronger version $A'' < 0$ of the standard c-theorem, $A'' \leq 0$. However for negative Λ in general positive A'' is allowed. The solutions written down above actually saturate the bound.

3. Spectrum of linearized fluctuations

3.1 Gravity on the AdS brane

Despite the absence of a massless graviton, we expect physics to be continuous and to react smoothly to small changes in the input parameters, like the brane tension. If we were to lose localized gravity by going from zero to tiny negative cosmological constant, this would mean that by the very fact that four-dimensional gravity exists, we could rule out an AdS_4 brane world. Similarly, one can argue that low energy physics on the brane should only probe the warp factor close to the brane, which is basically indistinguishable for the Minkowski and AdS cases, and hence should be insensitive to the eventual turn around in the warp factor. From a holographic perspective, if we probe sufficiently short distances on the brane, physics should not depend on the warp factor far away. This fact becomes even more striking when we heat up the brane universe to finite temperature. In this case a black hole will form in the bulk, and space time gets cut off at the black hole horizon, manifestly localizing gravity and hiding all the differences between the zero and negative cosmological constant scenario behind an event horizon.

Indeed, we argue that four-dimensional gravity is localized to the brane. We first argue that the propagator between two points that are sufficiently close relative to the scale set by the AdS_4 curvature will reduce to the Minkowski brane propagator.

In Minkowski space, one finds [10] that in the conformal "z-coordinates" in which the metric reads $ds^2 = e^{2A(z)}(g_{ij}dx^i dx^j - dz^2)$ the propagator can be written as

$$\Delta_{d+1}(x, z; x', z') = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} \Delta_p(z, z') \,, \tag{3.1}$$

where

$$e^{-2A}\left(\partial_z^2 + 3A'\partial_z - p^2\right)\Delta_p(z, z') = e^{-5A}\delta(z - z') \,. \tag{3.2}$$

In the AdS case, the integral in the Fourier transform will be replaced by a sum over discrete modes, whose spacing is of order $|\Lambda|$. So unless we want to study cosmological distance scales, the sum can still be approximated by an integral and then (3.2) tells us that the propagator for processes on the brane only depends on the warp factor in the vincinity of the brane.

Performing the coordinate change to the "z-coordinate", as we will work out later, the warp factor reads

$$e^A = \frac{L\sqrt{|\Lambda|}}{\sin\left((|z|+z_0)\sqrt{|\Lambda|}\right)}. \tag{3.3}$$

For small $|z|$, the sin can be expanded and we obtain to leading order in $|z|$

$$e^A = \frac{1}{1+\cos\left(z_0\sqrt{|\Lambda|}\right)\,|z|/L}, \tag{3.4}$$

which is identical to the flat space form $e^A = \frac{1}{|z|/L+1}$ up to a rescaling in L, so that the form of the differential equation stays unchanged. The higher order corrections are accompanied with powers of $|\Lambda| \sim e^{-2c}$ and hence are very small for near critical tension.

To complete this analysis, and demonstrate the correct polarization structure of the propagator, one would have to incorporate the effect of brane bending. This will be discussed in the mode analysis below. However, one can use the propagator analysis, as performed in ref. [11], to see that the correct gravitational potential is reproduced.

In [11], one explicitly solves for the trace and the longitudinal components of the metric perturbation in terms of the sources (rather than changing coordinates so that the brane is bent). These solutions then set up an "effective source term" for the TT-components, which includes contributions from the trace and longitudinal components of the metric. In terms of the trace reversed metric perturbation $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}$ this effective source with stress energy only on the brane leads in flat space leads to a linearized response

$$\begin{aligned}\bar{h}_{\mu\nu}(x) = -\frac{1}{16}\int d^4x' \Big\{ &\Delta_5(x,0;x',0)T_{\mu\nu}(x') - \\ &- \eta_{\mu\nu}\left[\Delta_5(x,0;x',0) - \frac{2}{L}\Delta_4(x,x')\right]\frac{T^\lambda_\lambda(x')}{4}\Big\}\end{aligned} \tag{3.5}$$

that contains an extra piece corresponding to the contribution of the extra scalar, that cancels the extra scalar contribution from the propagator of a massive mode. A similar analysis works in our AdS case with the scalar picking up a mass 4Λ, so that o ordinary gravity plus small corrections is reproduced.

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It is important to see explicitly how the above propagator, which was treated approximately in the above, is reproduced in a mode analysis. In the next section, we solve for the modes of linearized fluctuations around the brane, so that one can explicitly construct the propagator.

3.2 Analog quantum mechanics for transverse traceless modes

In this section, we study the spectrum of linearized gravity fluctuations around the dS_4 and AdS_4 solutions. We first consider the transverse-traceless (TT) modes, defined by

$$D^j h_{ij} = g^{ij}\, h_{ij} = 0\,, \tag{3.6}$$

where the perturbed metric is

$$ds^2 = e^{2A(r)}(g_{ij} + h_{ij})\, dx^i dx^j - dr^2\,, \tag{3.7}$$

and we have chosen axial gauge where

$$h_{\mu 5} = 0\,, \tag{3.8}$$

with $\mu = 0, 1, 2, 3, 5$. We will consider the non-TT modes in the next section, and in more detail in [12]. As suggested in [1], the TT modes are best analyzed by transforming Einstein's equations for the transverse traceless modes of the linearized fluctuations into the form of an analog quantum mechanics, where they satisfy a standard Schrödinger equation with a "volcano potential", which can be derived for arbitrary warp factor, see e.g. [2, 13].

Their linearized equation of motion reads, see e.g. [14, 15]

$$\left(\partial_r^2 + 4A'\partial_r - e^{-2A}(\Box_{4d} + 2\Lambda)\right) h_{ij} = 0\,. \tag{3.9}$$

We are looking for a mode corresponding to a 4d graviton, or more generally, a spin 2 excitation of mass m; that is

$$(\Box_{4d} + 2\Lambda) h_{ij} = m^2 h_{ij}\,, \tag{3.10}$$

where $\Box_{4d}$ is the four-dimensional covariant d'alembertian and m^2 the mass of the excitation. In the curved case, the constant shift by 2Λ is precisely the one needed in order to recover the equation of motion for a massless graviton (defined via the reduced number of polarization states) for $m^2 = 0$. Following the by now standard procedure, going to the conformal "z-coordinate" in terms of which the metric reads

$$ds^2 = e^{2A(z)}\left((g_{ij} + h_{ij})\, dx^i dx^j - dz^2\right). \tag{3.11}$$

and rescaling $H_{ij}(z) = e^{3A(z)/2}\, h_{ij}$, we obtain the Schrödinger equation of the analog quantum mechanics

$$\left(-\partial_z^2 + \frac{9}{4}A'(z)^2 + \frac{3}{2}A''(z)\right) H_{ij}(z) = m^2 H_{ij}(z)\,. \tag{3.12}$$

with the volcano potential

$$V(z) = \frac{9}{4}A'(z)^2 + \frac{3}{2}A''(z)\,. \tag{3.13}$$

3.3 The volcano potentials for dS and AdS

The change of coordinates to the conformally flat metric is given by

$$\begin{aligned} dS_4: \quad z(r) &= \frac{1}{\sqrt{\Lambda}}\,\mathrm{sgn}(r)\,\left\{\operatorname{arcsinh}\left(\frac{1}{\sinh\left(\frac{c-|r|}{L}\right)}\right) - z_0\sqrt{\Lambda}\right\}, \\ AdS_4: \quad z(r) &= \frac{1}{\sqrt{-\Lambda}}\,\mathrm{sgn}(r)\,\left\{\arcsin\left(\frac{1}{\cosh\left(\frac{c-|r|}{L}\right)}\right) - z_0\sqrt{-\Lambda}\right\}, \end{aligned} \tag{3.14}$$

where the parameter z_0 is defined as

$$\begin{aligned} dS_4: \quad z_0 &= \frac{1}{\sqrt{\Lambda}}\,\operatorname{arcsinh}\left(\frac{1}{\sinh(c/L)}\right), \\ AdS_4: \quad z_0 &= \frac{1}{\sqrt{-\Lambda}}\,\arcsin\left(\frac{1}{\cosh(c/L)}\right). \end{aligned} \tag{3.15}$$

Note that while in the dS case the z coordinate actually runs from $-\infty$ to $+\infty$ in the AdS case it only runs from $-(\pi/\sqrt{|\Lambda|} - z_0)$ to $(\pi/\sqrt{|\Lambda|} - z_0)$.

The resulting volcano potentials are

$$dS_4: \quad V(z) = \frac{9\Lambda}{4} + \frac{15}{4}\frac{\Lambda}{\sinh^2(\sqrt{\Lambda}(|z|+z_0))} - 3\,\coth\left(\sqrt{\Lambda}z_0\right)\,\sqrt{\Lambda}\,\delta(z)\,, \tag{3.16}$$

$$AdS_4: \quad V(z) = \frac{-9(-\Lambda)}{4} + \frac{15}{4}\frac{(-\Lambda)}{\sin^2(\sqrt{-\Lambda}(|z|+z_0))} - 3\,\cot\left(\sqrt{-\Lambda}z_0\right)\,\sqrt{-\Lambda}\,\delta(z)\,.$$

3.4 The de Sitter brane spectrum

The volcano potential for the deSitter case is plotted in figure 3. The general features of this potential are very similar to those of the flat space volcano of figure 2. Both potentials have a normalizable zero mode trapped in the delta potential, whose wave function is given by the warp factor $e^{3A(z)/2}$ and decays like $1/z^3$. In the Minkowski case this massless graviton is a marginal bound state and the continuum of KK modes starts at $m = 0$. In the dS case, since at infinity the potential approaches a constant value $\frac{9}{4}\Lambda$, the onset of the continuum is only at $m^2 = \frac{9}{4}\Lambda$ and the graviton is separated by a finite mass gap from the continuum [15].

3.5 The embedding of AdS_4

The Anti-deSitter volcano potential, as shown in figure 4, looks quite different. As remarked above, the z-coordinate only stretches over the finite range from $-(\pi/\sqrt{|\Lambda|} - z_0)$ to $(\pi/\sqrt{|\Lambda|} - z_0)$. The potential diverges at the boundaries, so that we effectively

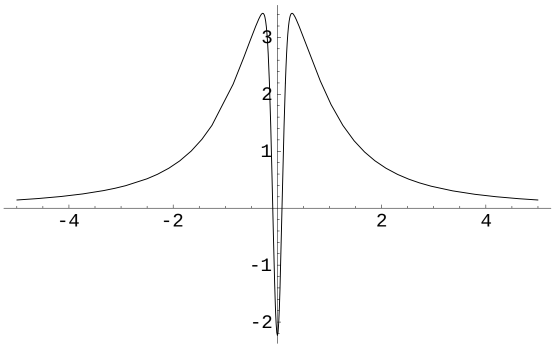

Figure 2: The Minkowski volcano potential.

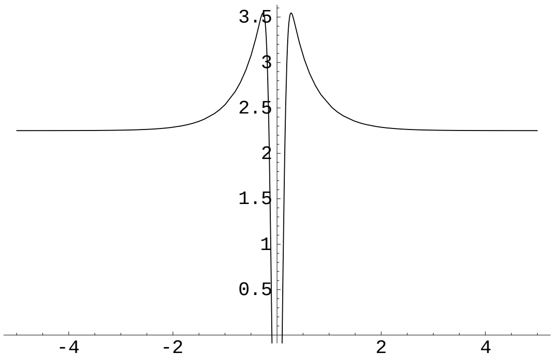

Figure 3: The deSitter volcano potential.

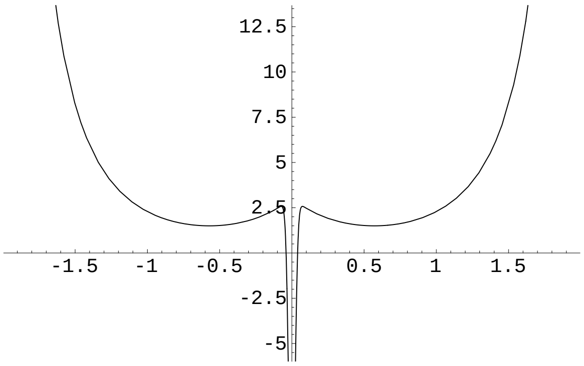

Figure 4: The Anti-deSitter volcano potential.

create a box. As we tune the effective 4D cosmological constant from positive to negative values, the flanks of the volcano move down from $\frac{9}{4}\Lambda$ to zero and then the potential "curls" in and we are left with a discrete spectrum of modes without a zero mode. The would be zero mode wavefunction diverges together with the warp factor $e^{A(z)}$ at the boundaries of the box.

Before we solve for the discrete spectrum of the quantum mechanics explicitly let us discuss why we see this behavior. The crucial point is that while the region of space bounded by the worldvolume of a single positive tension dS or Minkowski brane does not contain any part of the boundary of AdS_5, for the AdS brane this is not the case.

Figures 5 and 6 summarize the difference between the flat and deSitter universes on the one hand and the AdS universe on the other hand. For the former the RS cutting and pasting procedure leads to a space that ends in a horizon on both sides. In the AdS_4 case even though n the space-time we keep it looks close to the brane like we are moving towards the horizon (the warp factor decreases) eventually we have to go all the way out to the boundary. While for the flat brane, gravitons sent out from the brane never come back but fall through the horizon, for the AdS_4 brane

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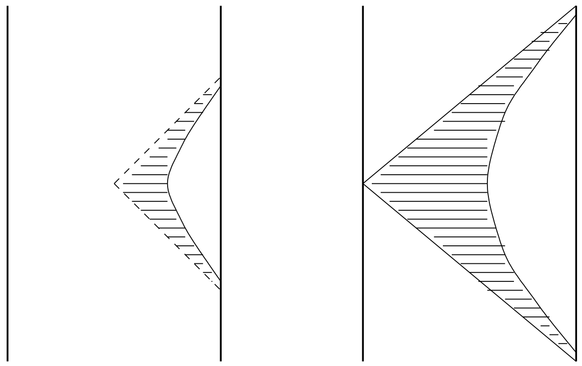

Figure 5: Schematics of the Penrose diagrams of the dS and Minkowski. The spacetime one is instructed to keep is shaded. Since the branes are accelerated, they have their own horizon, only for the Minkowski brane does this coincide with the Poincare patch horizon. The spacetime one wants to keep is between the brane and the horizon. The dS brane really is a full hyperboloid, the part drawn corresponding to the static slice of dS. In the case of the AdS brane the brane falls through the horizon of the Poincare patch. It is more useful to study the embedding in terms of an constant time slice through global AdS.

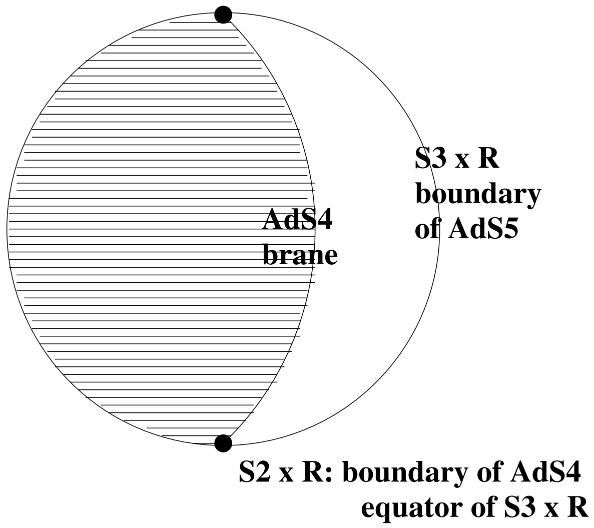

Figure 6: Constant time slice through AdS_5, with the AdS_4 brane included and again the region of spacetime we are instructed to keep shaded. Global time on the brane is global time in AdS_5, the picture does not evolve in time.

some eventually bounce back from the boundary and make it back to the brane. Effectively the 5d spacetime is a box and the gravity spectrum is discrete, at least when considered with respect to the hamiltonian generating global time translations.

In [2] a coordinate transformation was given for the Poincare patch of AdS_5 from written as a warped product with an AdS_4 Poincare patch to the standard form. One can extend this coordinate transformation to the full global solution:

$$ds^2 = L^2 \cosh^2\left(\frac{c-r}{L}\right) ds_4^2 - dr^2 \tag{3.17}$$

where

$$ds_4^2 = \frac{1}{\cos^2(\rho)} \left(dt^2 - d\rho^2 - \sin^2(\rho)\, d\Omega_2^2\right). \tag{3.18}$$

Applying the following change of variables

$$\begin{aligned} \tan(\tilde{\rho}) = \coth\left(\frac{c-r}{L}\right)\tan(\rho)\,, \qquad \cosh(\tilde{r}) = \frac{\cosh(\frac{c-r}{L})}{\cos(\rho)}\,, \\ \tilde{t} = t\,, \qquad \tilde{\Omega}_2 = \Omega_2 \end{aligned} \tag{3.19}$$

we end up with

$$ds^2 = L^2\left(\cosh^2(\tilde{r})\, dt^2 - d\tilde{r}^2 - \sinh^2(\tilde{r}) d\tilde{\Omega}_3^2\right), \tag{3.20}$$

where $\tilde{\Omega}_3$ is the 3-sphere metric on $\tilde{\rho}$ and $\tilde{\Omega}_2$, $d\tilde{\Omega}_3^2 = d\tilde{\rho}^2 + \sin^2(\tilde{\rho})\tilde{\Omega}_2^2$. This can be recognized as global AdS_5, see for example [16].[2] Using this change of variables, it is easy to see, that an $r = const.$ surface is embedded in global AdS_5 as indicated in figure 6.

Note that this implies an interesting realization of holography. Gravity in the bulk of our space should be dual to a theory living on the full boundary, which is composed of two pieces, the AdS_4 brane and the $S^3/\mathbb{Z}_2$ half-boundary of the original AdS_5, with two different field theories living on them. They touch along their common boundary. According to AdS/CFT lore, any theory in AdS can be completely encoded in its boundary data, so that in the end all AdS_4 bulk physics should be captured by the CFT on the disk with the brane physics encoded in the boundary data of the disk. Instead of a CFT with momentum somehow cutoff, we get a CFT with spatial cutoff (that is formulated on a disk). The reduction of the symmetry from SO(4, 2) to SO(3, 2) corresponds to the well-known fact that this is the subgroup of conformal transformations that leaves a boundary invariant.

3.6 The discrete spectrum in the volcano potential

We now solve the quantum mechanics problem with the box-like volcano potential (3.16). In order to solve the spectrum with a delta function at the bottom of a box, we first find the eigenfunctions in the box, and then look for the right linear combinations to satisfy the jump imposed on the first derivative of the wavefunction due to the delta function.

[2] Note that while ρ runs from 0 to $\pi/2$ and r from $-\infty$ to ∞, $\tilde{\rho}$ runs from 0 to π and $\tilde{r}$ from 0 to $+\infty$. When we take r to zero, $\tilde{\rho}$ has to go to $\pi/2$. As r goes negative, on the other side $\tilde{\rho}$ now explores the region between $\pi/2$ and π

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We first consider the zero tension case, where formally the delta contribution vanishes.[3] The turn around point c in the warp factor (2.3) goes to zero, the solution becomes just $A = \log(L\sqrt{\Lambda}\cosh\frac{r}{L})$, and we recover pure AdS_5. The parameter z_0 in (3.16) becomes $\sqrt{|\Lambda|}z_0 = \pi/2$ and the resulting potential is just

$$V(z) = \frac{-9(-\Lambda)}{4} + \frac{15}{4}\frac{(-\Lambda)}{\cos^2\left(\sqrt{-\Lambda}z\right)} \tag{3.21}$$

with z running from $-\pi/2$ to $\pi/2$. Rescaling $z \to \sqrt{-\Lambda}z$, Schrödinger's equation with energy eigenvalue $-E/\Lambda$ in the potential (3.21) can be solved explicitly and the resulting wavefunction is

$$\begin{aligned}\psi(z) = c_1 \cdot {}_2F_1\left(-\frac{3}{4} - \frac{\sqrt{9+4E}}{4}, \frac{5}{4} - \frac{\sqrt{9+4E}}{4}; 1 - \frac{\sqrt{9+4E}}{2}; \frac{1}{\cos^2 z}\right) \times \\ \times \cos^{\sqrt{9+4E}/2} z + c_2 \cdot {}_2F_1\left(-\frac{3}{4} + \frac{\sqrt{9+4E}}{4}, \frac{5}{4} + \frac{\sqrt{9+4E}}{4}; 1 + \frac{\sqrt{9+4E}}{2};\right. \\ \left.\frac{1}{\cos^2 z}\right) \cos^{-\sqrt{9+4E}/2} z\,. \end{aligned} \tag{3.22}$$

In order to not diverge on either side of the box we need $c_2 = 0$ and

$$E = n\,(n+3) \qquad \text{where} \qquad n = 1, 2, 3, \ldots\,. \tag{3.23}$$

This spectrum indeed agrees with the spectrum of gravity fluctuations of pure AdS_5, as can be seen to be the case by group-theoretic considerations. Unitary representations of the AdS_5 group are classified (for a recent discussion see e.g. [17]) by the eigenvalue of the global time hamiltonian E_0, which is the generator of the SO(2) subgroup of SO(4) $\times$ SO(2) $\subset$ SO(2, 4), and SO(4) spins j_1 and j_2 for the groundstate from which the infinite dimensional representation is built by raising operators. The massless graviton (where masslessness is defined via the smaller number of polarization states) has $E_0 = 4$ and $j_1 = j_2 = 1$. The analysis of the analog quantum mechanics amounts to decomposing the SO(4, 2) representation under the SO(3, 2) subgroup that is manifest in the warped product based on AdS_4. As in 5d, these are classified by E_0 and s, where E_0 in 4d is identified with E in 5d (the SO(2) subgroup corresponding to global time translations of SO(4, 2) is the same as in SO(3, 2)) and s is the SO(3) spin obtained by tensoring together j_1 and j_2. The Casimir now reads

$$C_2(E_0, s) = E_0(E_0 - 3) + s(s+1) \tag{3.24}$$

and a massless representation obeys

$$E_0 = s + 1\,. \tag{3.25}$$

[3]If one actually wants to analyse the spectrum of an almost tensionless brane, the spectrum will be the same, but every other mode is projected out by the boundary condition at the brane. By just omitting the delta function we are really looking at "no brane" instead of a zero tension brane.

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So the apropriate definition of mass for a spin 2 representation is

$$m^2 = C_2(E_0, 2) - C_2(3, 2) = E_0(E_0 - 3)\,. \tag{3.26}$$

The single massless graviton decomposes into a tower of spin 0,1 and 2 representations in four dimensions. The masses of the spin 2 excitations are

$$m^2 = E_{0,n}(E_{0,n} - 3) \qquad \text{with} \qquad E_{0,n} = 4, 5, 6, \ldots \tag{3.27}$$

which obviously agrees with the spectrum (3.23) found via the Schrödinger equation.

Upon including the brane the solution in terms of the linear combination of hypergeometric functions becomes quite complicated and we chose to study it numerically.[4] A more detailed analysis of the spectrum will appear elsewhere. Let us briefly exhibit the crucial features of the spectrum as obtained via a numerical solution of Schrödinger's equation using the shooting method.

In order to find the eigenvalues and wavefunctions for non-zero tension, we use

$$\frac{\psi'}{\psi} = -3\tan(z_0) \tag{3.28}$$

as required by the delta function in the potential as the start condition for wavefunction on the brane and then numerically solve Schrödinger's equation outwards. For generic energy values, the would-be wavefunction will diverge at the boundary. Eigenvalues are the values of the energy for which this expression goes to zero at the boundary. Of course, this is never really zero in any numerical approximation. The shooting method relies on the fact that (for any reasonable potential) the would be wavefunction diverges to (say) +infinity if you are below an eigenvalue and then will diverge to -infinity above the eigenvalue (and then vice versa for the next eigenvalue). If one plots the wavefunction at the boundary as a function of energy, the zeroes of this function are the eigenvalues.

Figures 7, 8 and 9 show the spectrum of low lying modes as we increase the tension. For small tension, we keep every other mode of the pure AdS_5 analysis. As we increase the tension, one mode comes down to mass[5] $m^2 \ll \Lambda$, while all the other modes stay at $m^2 = \mathcal{O}(\Lambda)$. As we approach the critical tension, the potential becomes "two copies" of the pure AdS_5 potential pasted together along a delta function. Indeed we have all the heavy modes of the AdS_5 analysis at $m^2 = (n-3)n\Lambda$ and in addition a very light mode trapped in the delta function. In the critical limit the trapped very light mode becomes the trapped massless graviton, while the densely spaced excited modes become the continuum of KK modes. Figures 10 and 11 show the wavefunctions of the almost massless mode and of a highly excited mode. Comparision with the shape of the zero mode and the KK-modes in the RS

[4] For an analytic treatment see [18].

[5] A more detailed numerical analysis [19] or an analytic treatment relying on the factorization of the hamiltonian [18] both show that the actual behavior for the almost zero mode is $m^2 \sim \frac{3}{2}\Lambda^2$.

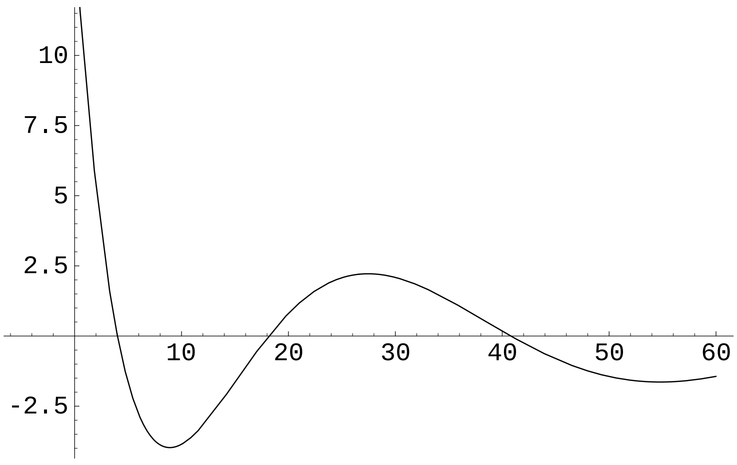

Figure 7: Spectrum at zero tension, $\sqrt{|\Lambda|} z_0 = \pi/2$.

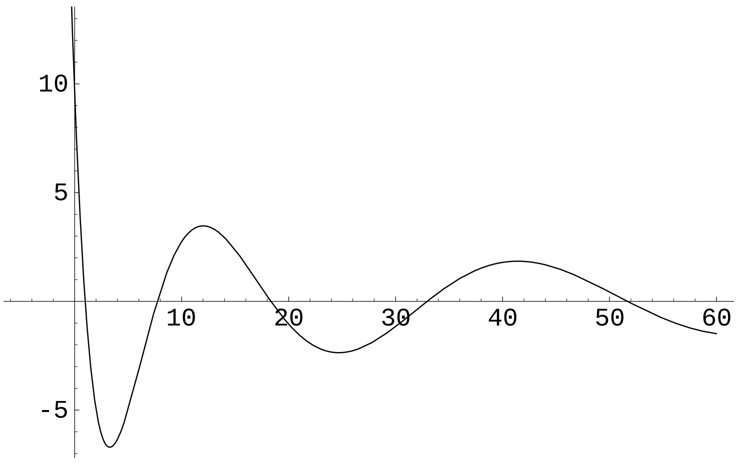

Figure 8: Spectrum at $\sqrt{|\Lambda|} z_0 = \pi/2 - 0.7$.

scenario show indeed that in the flat space limit, the former approaches the zero mode localized on the brane, while the latter approaches a highly excited KK mode, oscillating without feeling the brane at all. The low KK modes also look as expected: oscillating in the bulk but suppressed on the brane.

As we decrease the tension, the wavefunction of the almost zero mode gets broader and loses its amplitude enhancement over the KK modes, as is studied in detail in [19]. Therefore, we would never actually be able to see four-dimensional AdS gravity and at the same time a significant deviation due to the graviton mass. For small Λ, the mass is tiny, and by the time we reach distance scales of order $1/m$ where the mass of the graviton should become significant, all excitations have died off anyway due to the gravitational potential of AdS. When we go to $\Lambda \to 1/L^2$, which is the maximal value we can reach for very small tension branes, the mass

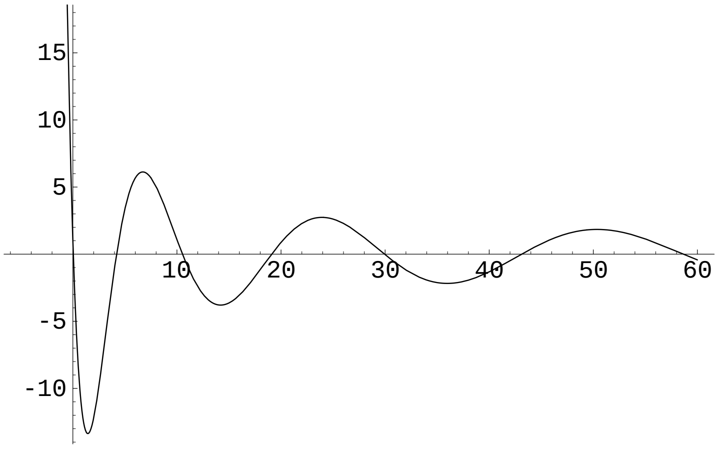

Figure 9: Spectrum at $\sqrt{|\Lambda|}z_0 = \pi/2 - 1.4$.

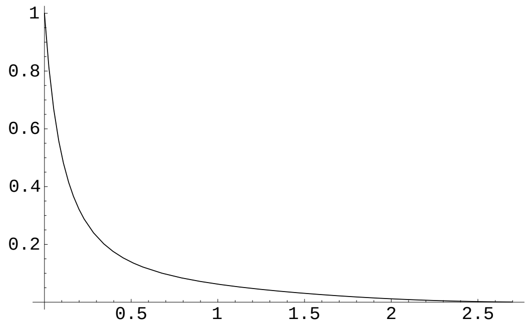

Figure 10: Wavefunction of the almost massless mode, E=0.0419 at $\sqrt{|\Lambda|}z_0 = \pi/2 - 1.4$.

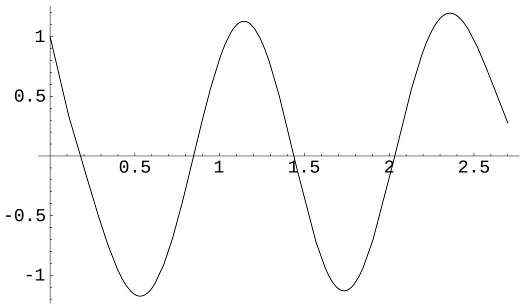

Figure 11: Excited mode at $\sqrt{|\Lambda|}z_0 = \pi/2 - 1.4$ and E=30.

of the almost zero mode goes like 4Λ and hence one should now see it contributing on the same footing as the 4d curvature effects. However since in this limit, the wavefunction is no longer bound to the brane and the almost zero mode has a mass comparable to the other excited modes, it really becomes one of the KK-modes and one no longer has four-dimensional gravity at all.

4. The non-TT modes

So far we have only analyzed the spectrum of TT fluctuations. We have seen that in the AdS case, we get a massive spin 2 almost zero mode and the usual massive KK tower. This naively seems inconsistent, since the massive mode has extra polarization states that should contribute to the propagator. However, we expect a smooth flat space limit. In order to complete the analysis, we have to study the non-transverse-traceless modes as well. We will argue that although the additional modes can be gauged away up to a brane bending effect, they nonetheless contribute to the propagator, effectively reducing the degrees of freedom from 5 to 2. This will be discussed further in ref. [12].

We first analyze the general form of a solution to the equations of motion and then compare this with the residual gauge transformations. We still write the perturbed metric as

$$ds^2 = e^{2A(r)}(g_{ij} + h_{ij})\, dx^i dx^j - dr^2\,, \tag{4.1}$$

in axial gauge

$$h_{\mu 5} = 0\,, \tag{4.2}$$

where $\mu = 0, 1, 2, 3, 5$, but do not restrict to TT components only. We first exhibit the general form of the solution to the bulk equations of motion.

The first order equations of motion require[6] vanishing of (where $h = g^{ij} h_{ij}$):

$$\begin{aligned}
55 &: (e^{2A} h')'\,,\\
5i &: -\frac{1}{2}\nabla_i h' + \frac{1}{2}\nabla^k h'_{ik}\,,\\
ij &: e^{2A}\left(\frac{1}{2}h''_{ij} + 2A' h'_{ij}\right) - \frac{1}{2}\nabla^2 h_{ij} + \frac{1}{2} g_{ij} e^{2A} A' h' -\\
&\quad - \frac{1}{2}\nabla_i \nabla_j h + \frac{1}{2}\left(\nabla^k \nabla_i h_{jk} + \nabla^k \nabla_j h_{ik}\right) + 3\Lambda h_{ij}\,.
\end{aligned} \tag{4.3}$$

In order to manipulate these expressions, the following identities for covariant derivatives in dS and AdS are useful:

$$\begin{aligned}
R_{ijkl} &= \Lambda(g_{il} g_{jk} - g_{ik} g_{jl})\,,\\
R_{ij} &= g^{jl} R_{ijkl} = -3\Lambda g_{ij}\,,\\
\nabla_i \nabla_j \nabla_k f - \nabla_j \nabla_i \nabla_k f &= \Lambda(g_{jk}\nabla_i - g_{ik}\nabla_j) f\,,\\
\nabla_i \nabla_k \nabla^k f - \nabla^k \nabla_i \nabla_k f &= 3\Lambda \nabla_i f\,,\\
\nabla_i \nabla_j \nabla^k \nabla_k f - \nabla^k \nabla_k \nabla_i \nabla_j f &= (8\Lambda \nabla_i \nabla_j - 2\Lambda g_{ij} \nabla^2) f\,,\\
\nabla^k \nabla_i \nabla_j \nabla_k f - \nabla^k \nabla_k \nabla_i \nabla_j f &= (-\Lambda g_{ij} \nabla^2 f + \Lambda \nabla_i \nabla_j f)\,.
\end{aligned} \tag{4.4}$$

[6] In the ij equations we used the zeroth order equations of motion to eliminate the terms involving the stress energy tensor on the right hand side of Einstein's equations as in [2, 20].

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From the 55 equations it follows immidiately that h_{ij} can be written as

$$h_{ij} = h_{ij}^{\mathrm{TT}} + G\, f_{ij} + d_{ij}\,, \tag{4.5}$$

where h_{ij}^{TT} are the TT components of the fluctuation which we studied in great detail before, f_{ij} and d_{ij} are functions of the 4d space-time only and G is defined as[7]

$$G = \int^r e^{-2A(\tilde r)} d\tilde r\,. \tag{4.6}$$

Now from the $i5$ we find

$$\nabla_i f_k^k = \nabla^k f_{ik}\,. \tag{4.7}$$

Using as an ansatz

$$f_{ij} = \nabla_i \nabla_j \alpha - g_{ij}\beta \tag{4.8}$$

we see that (4.7) implies

$$f_{ij} = (\nabla_i \nabla_j - \Lambda g_{ij})\Phi\,. \tag{4.9}$$

Φ is what we would like to identify as the radion field. Plugging this into the ij equation, one finds after a tedious calculation using (4.4) and the following property of our warpfactors

$$A' + G\Lambda = -\frac{\lambda}{3} \tag{4.10}$$

that the full system is solved by

$$h_{ij} = h_{ij}^{\mathrm{TT}} + 2G\,(\nabla_i \nabla_j - \Lambda g_{ij})\Phi - \frac{2}{3}\lambda g_{ij}\Phi\,, \tag{4.11}$$

where Φ has to satisfy the following equation of motion:

$$\nabla^2 \Phi = 4\Lambda\Phi \tag{4.12}$$

and the TT modes h_{ij}^{TT} are determined by the analog quantum mechanics we analyzed before. The boundary condition at the brane demands

$$h'_{ij} = 0 = h_{ij}^{\mathrm{TT}'} + 2e^{-2A}(\nabla_i \nabla_j - \Lambda g_{ij})\Phi\,. \tag{4.13}$$

Now let us proceed to analyze the residual gauge transformations that remain after fixing axial gauge. Under gauge transformations that take $x^\mu \to x^\mu + \xi^\mu$, the fluctuations transform as

$$\begin{aligned} h_{55} &\longrightarrow h_{55} - 2e^{-2A}\xi^{5'}\,, \\ h_{ij} &\longrightarrow h_{ij} + (\nabla_i \xi_j + \nabla_j \xi_i) + 2A' g_{ij}\xi^5\,, \\ h_{i5} &\longrightarrow h_{i5} - e^{-2A}\nabla_i \xi^5 + g_{ij}\xi^{j'}\,, \end{aligned} \tag{4.14}$$

[7]This of course only defines G up to a constant, which can be absorbed into d_{ij}. To be precise, we will always choose 0 as the lower bound on the integral.

where we pulled out a factor of e^{2A} from the h_{55} and h_{i5} component as we did for the h_{ij} fluctuation in (4.1) and G is defined as in (4.6) above. In order to preserve the axial gauge (4.2) we require that h_{55} and h_{i5} stay zero, leaving us with the residual gauge transformations given by

$$\begin{aligned}\xi_5 &= \epsilon^5(x)\,,\\ \xi_i &= G\nabla_i\epsilon^5 + \epsilon_i(x)\,.\end{aligned} \tag{4.15}$$

As in [10], the ϵ^i gauge transformations are needed in order to gauge away the longitudinal components of the graviton fluctuations, which we already neglected by our ansatz for h_{ij}. Using (4.10) one can see that the residual gauge transformations (4.15) preserving axial gauge can indeed be used to gauge away the radion component Φ in the full solution (4.11) by choosing $\epsilon^5 = -\Phi$.

In the presence of sources on the brane, the ϵ_5 gauge transformation leads, as in [10], to a bent brane in the new gauge and to a brane bending contribution to the propagator, proportional to the propagator of a scalar field of mass 4Λ. This scalar is clearly not one of the physical modes, since we can gauge it away everywhere where there is no stress energy. Hence it does not correspond to a propagating ghost. In this respect our setup differs crucially from the quasilocalized gravity model of [21] that was analyzed in a very similar fashion in [22]–[26]. There, due to the additional boundary condition on the second brane the radion cannot be gauged away [27] and represents a real ghost. In the critical tension limit, the bending contribution is precisely what is needed to smoothly match on to the standard massless 4d graviton propagator. The absence of a Veltman Van Dam discontinuity [28, 29], as the mass of the graviton goes to zero is consistent with the findings of [30][8] that states that no such discontinuity exists in AdS space as long as the mass m^2 goes to zero faster than Λ. This should not come as a surprise, since we preserved the gauge invariance corresponding to 4d diffeomorphisms despite the fact that we are dealing with a massive graviton. As will be explained in [12], these additional gauge invariances can still be used to eliminate 3 of the polarization modes.

5. Conclusions

In this paper, we have demonstrated that localization of gravity applies, even for the AdS_4 brane for which there is no normalizable zero mode. Although expected from the vantage point of locality and holography, it is interesting that a massive graviton generates four-dimensional gravity.

Our result differs substantially from quasi-localization models, that also had a turn-around in the warp factor that modifies gravity on long distance scales. In

[8] After completing this work, we became aware of [31], which obtains a similar result to [30].

the existing theories, there was a ghost when four-dimensional gravity is reproduced. Furthermore, those theories required unphysical matter in order to violate the C-theorem.

Our theory should have interesting implications for holography. A complete holographic description of the setup can be given in terms of a CFT living on the disk that is the left over part of the true AdS_5 boundary. In this description, all of the physics on the brane is reduced to correlators on the common boundary of the disk and AdS_4. This description is however not appropriate to study local physics on the brane. For latter purpose it should be more useful to split the bulk excitations into two sets, one dual to a CFT on the true boundary, the other one to a CFT on the brane, but the precise procedure is unclear. The mass of the almost zero mode is attributable to the long distance behavior of the warp factor which is sensitive to boundary physics; presumably the mass can be understood as a consequence of the boundary CFT. Understanding how the theory on the brane is reproduced by CFT physics remains a challenge.

The much more general phenomenon of localization demonstrated in this paper might have other implications. Compactification of extra-dimensional spaces is not essential to reproduce four-dimensional physics; a region of space which localizes gravity is sufficient. Furthermore, the current no-go theorems about localization in supergravity theories e.g. [32, 33, 34] relying on the asymptotic behavior of the geometry do not necessarily apply. It is important to know whether this physics can be realized in string theory.

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